



21.04.2019 – Week 11

Fatigue test

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Outline

- Introduction.
- Stresses cycles.
- Fatigue test apparatus.
- Failure modes.
- S-N curves.
- Soderberg & Goodman lines
- Solved example.
- Smith diagram.

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Introduction

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Introduction

A phenomenon which results in the sudden fracture of a component after a period of cyclic loading in the elastic regime.

Most structural assemblages are subject to variation in applied loads, causing fluctuations in stresses.

If the fluctuating stresses are large enough, even though the stress is less than static stress, failure may occur when the stress is repeated.

This type of failure is called a *fatigue* failure.

Fatigue failures occur as a result of material cracks due to repeated (cyclic) stresses or, in fact, strains.

Fatigue failures are often referred to as progressive fractures.

Fatigue

Fatigue is due to the repeated loading and unloading.

When a material is subjected to a force acting in different directions at different times it can cause cracking. In time this causes the material to fail at a load that is much less than its tensile strength, this is fatigue failure. Vibration for example is a serious cause of fatigue failure.

Fatigue

Fatigue can be prevented with good design practice.

1. A smooth surface finish reduces the chance of surface cracking.
2. Sharp corners should be avoided.
3. Corrosion should be avoided as this can cause fatigue cracks.

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Stresses cycles

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Stress cycles

Stress cycles are classified into three main types:

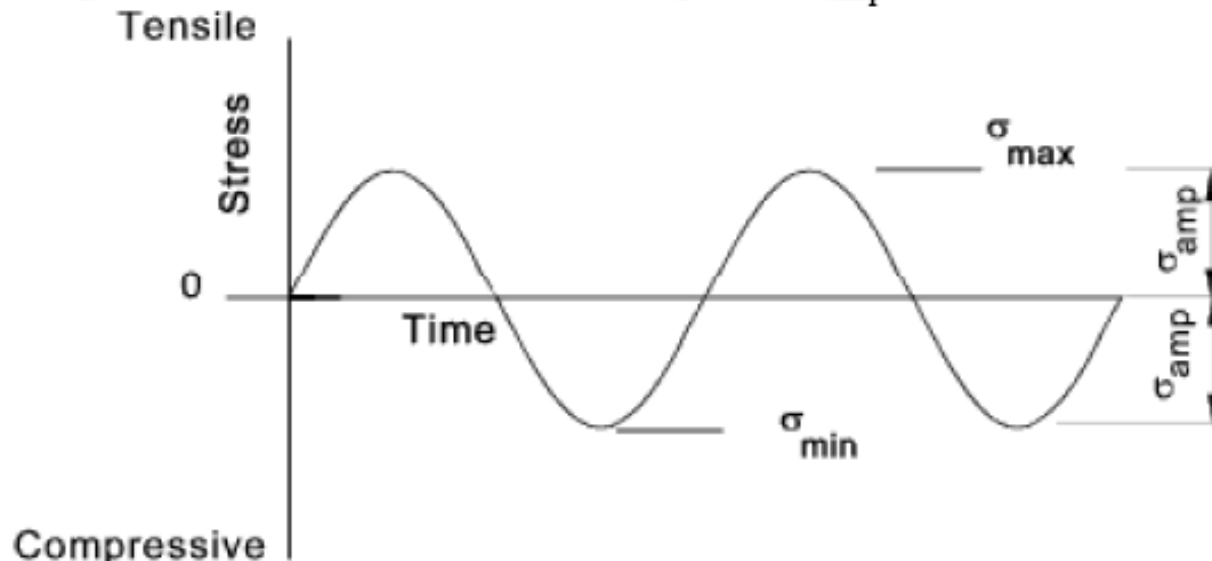
1. Alternating /Reversing Stress/ **Complete Reversing Cycle**

In determining fatigue stress levels using standard test equipment the test specimens are subject to alternative/reversed stress levels.

The cyclic stress varies from σ_{\max} tensile to σ_{\min} compressive (both are equal in magnitude).

The mean stress = 0.

The fatigue strength from the S-N curve, being the σ_{amp} value at failure.

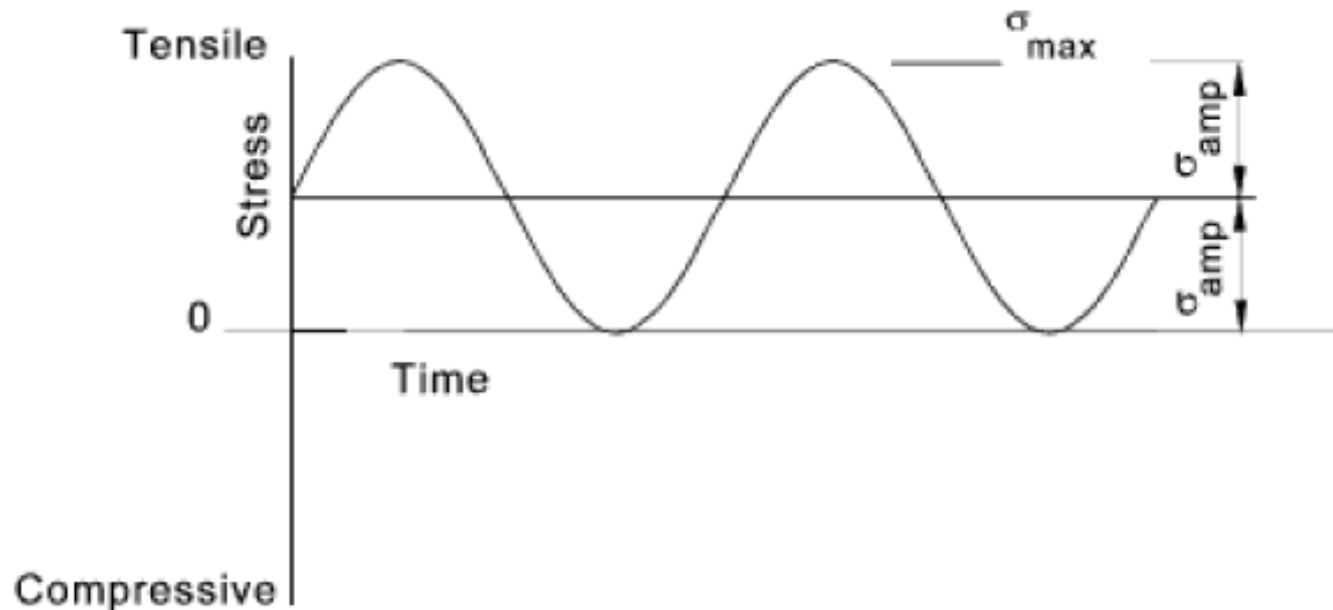


Stress cycles

2. Repeated Stress

In the repeated stress loading condition the stress varies between zero and a maximum tensile (or compressive) stress in a cyclic manner.

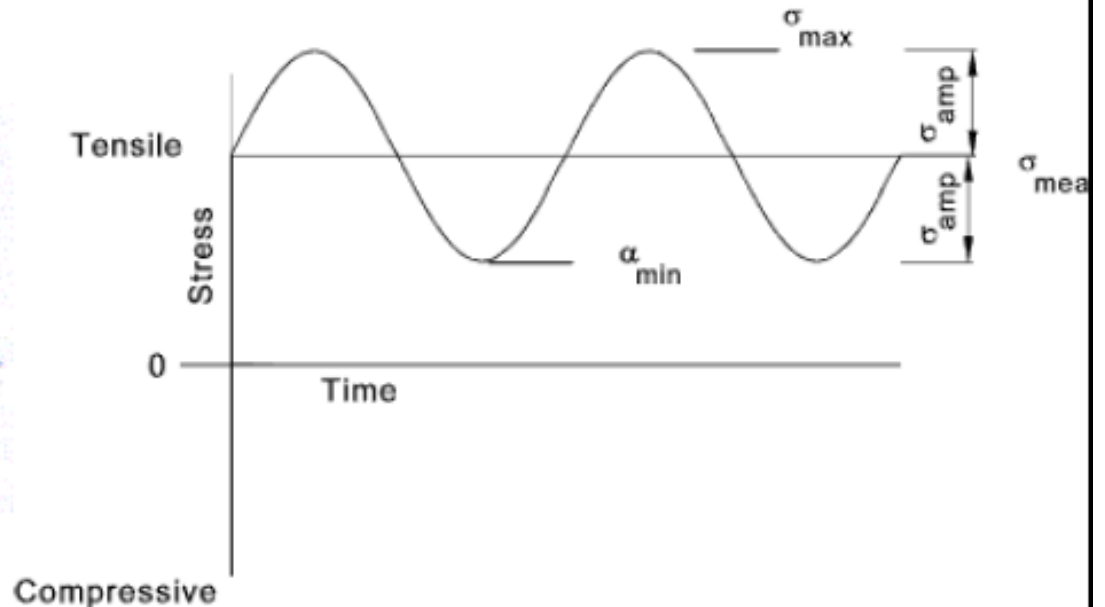
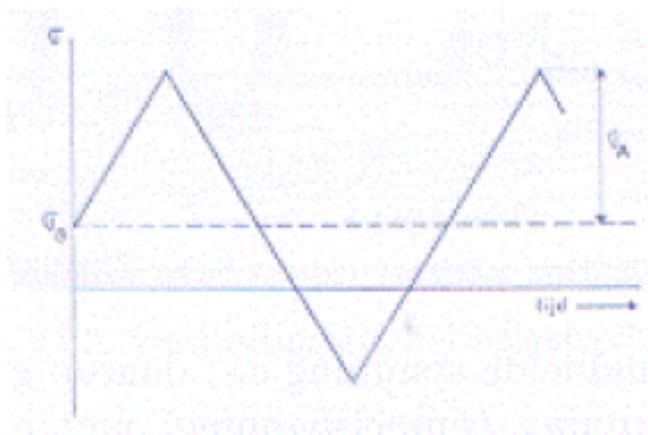
The cyclic stress varies from σ_{\max} tensile (or compression) to Zero. The minimum stress = 0.



Stress cycles

3. Combined steady and cyclic stress

A stress condition normally experienced in practice is a cyclic stress imposed on a steady load stress.



Notations

The maximum stress		$= \sigma_{\max}$
The minimum stress		$= \sigma_{\min}$
The mean stress	σ_{mean}	$= (\sigma_{\max} + \sigma_{\min}) / 2$
The stress amplitude	σ_{amp}	$= (\sigma_{\max} - \sigma_{\min}) / 2$ $= (\sigma_{\max} - \sigma_{\text{mean}})$ $= (\sigma_{\text{mean}} - \sigma_{\min})$
The stress Ratio	R	$= \sigma_{\min} / \sigma_{\max}$
The stress range		$= (\sigma_{\max} - \sigma_{\min})$

Characterization of Sinusoidal Cycles

σ_{\max} = maximum stress

σ_{\min} = minimum stress

σ_a = amplitude of alternating component

σ_m = midrange component

σ_s steady (Static) component

σ_r = range of stress = $|\sigma_{\max} - \sigma_{\min}|$

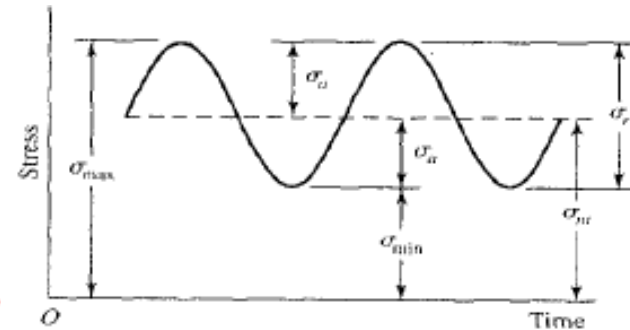
$$\sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2}$$

..... (4a)

$$\sigma_a = \frac{\sigma_{\max} - \sigma_{\min}}{2}$$

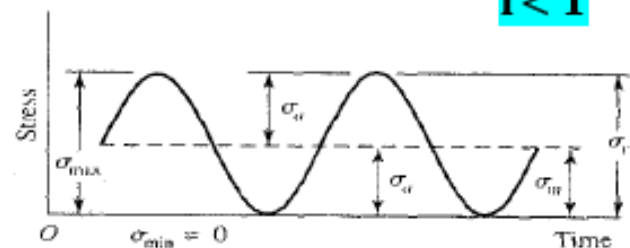
$r = \frac{\sigma_{\min}}{\sigma_{\max}}$ is the stress ratio or cycle ratio

$A = \frac{\sigma_a}{\sigma_m}$ is the amplitude ratio



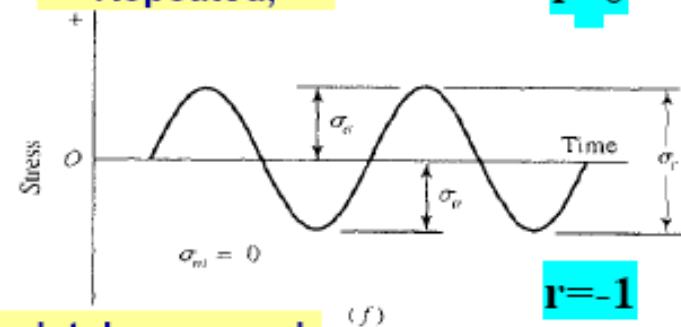
Fluctuating, (d)

$r < 1$



Repeated, (e)

$r = 0$

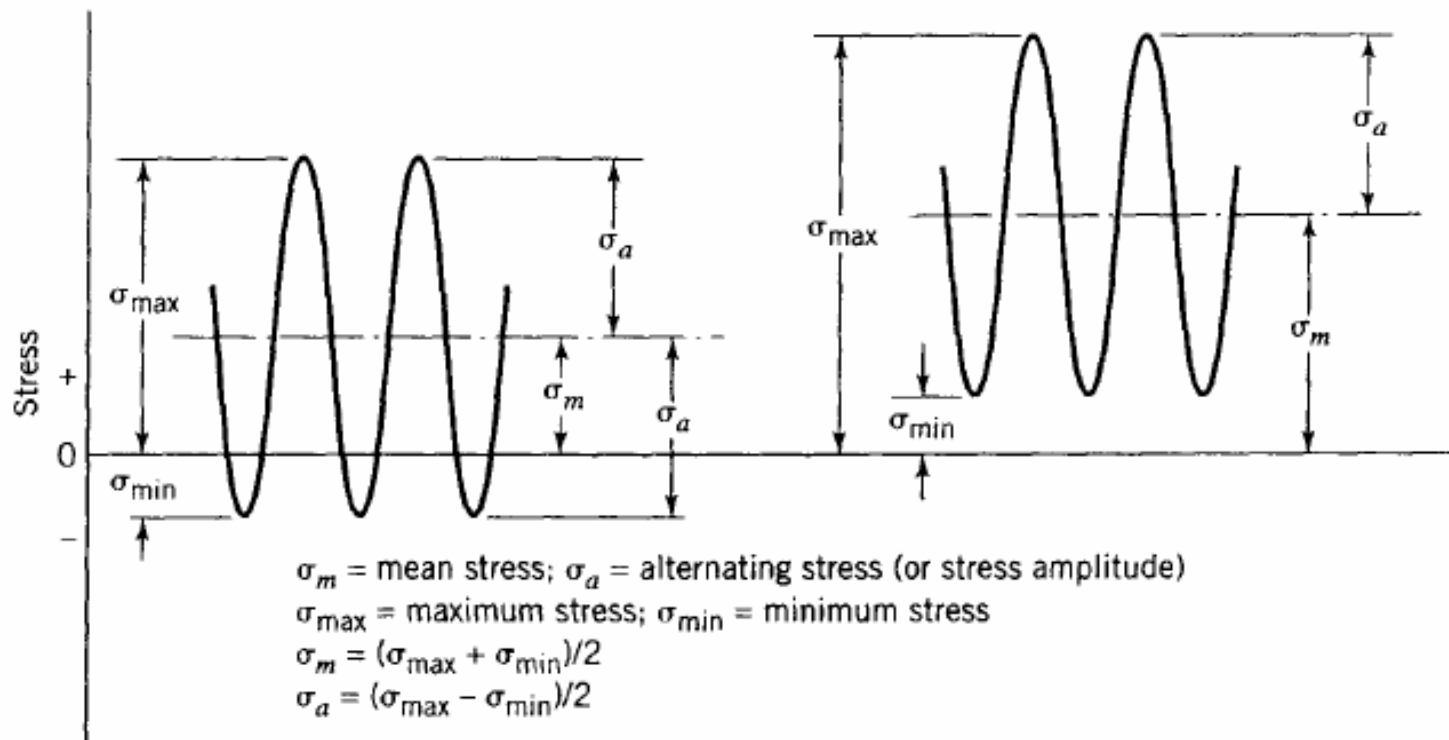


Completely reversed (f)

$r = -1$

Characterization of Sinusoidal Cycles

A fluctuating stress is usually characterized by its **mean (σ_m)** and **alternating (σ_a) stress components**, and sometimes **maximum and minimum stresses (σ_{\max} & σ_{\min})** are also used. All four of these quantities are defined in Fig.4-3.

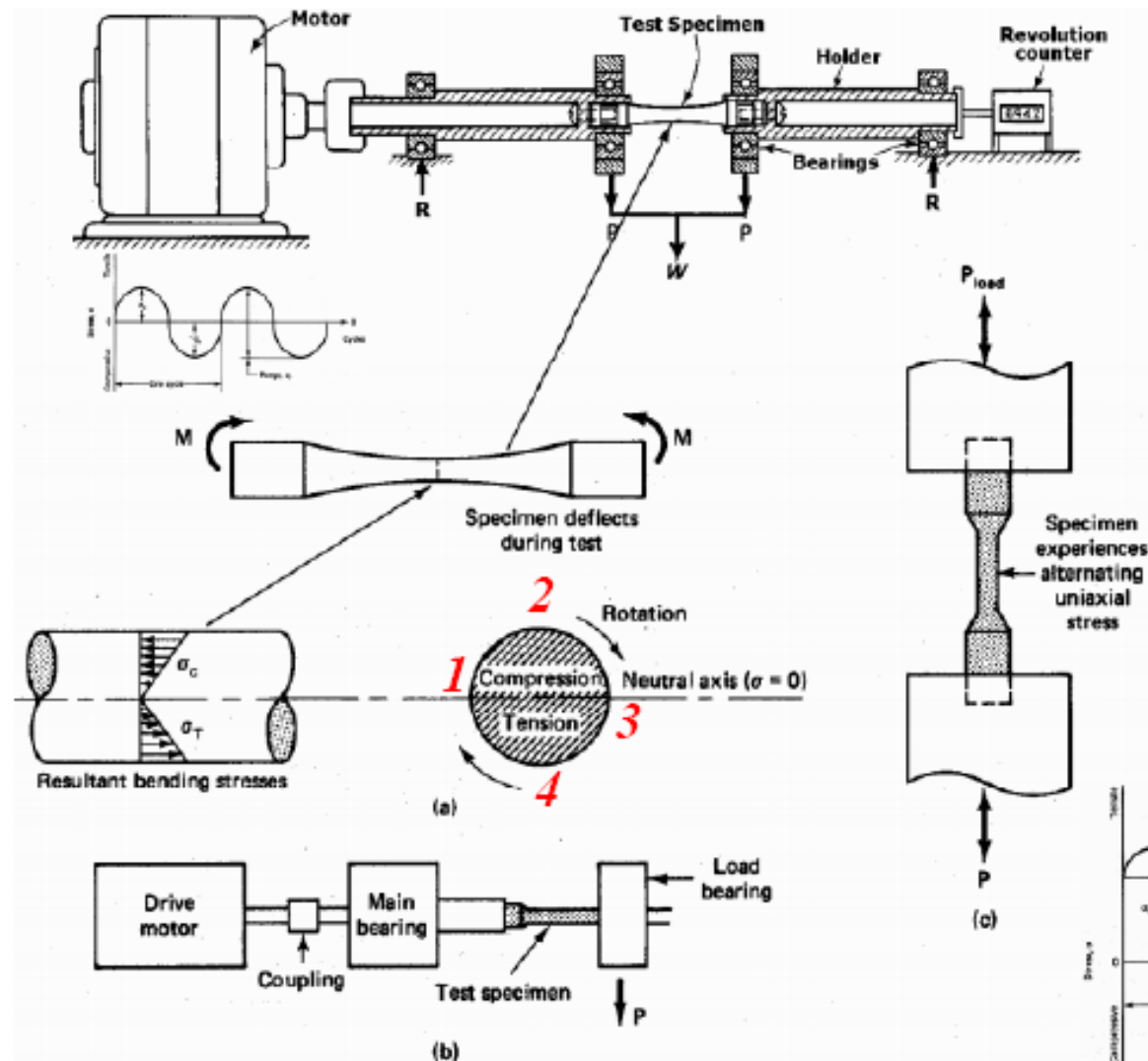


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Fatigue test apparatus

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Fatigue test apparatus

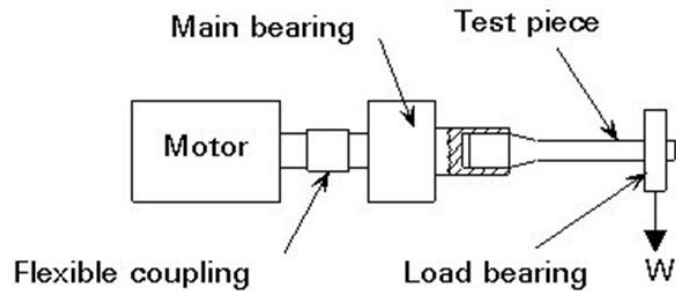


(a) the rotating bending

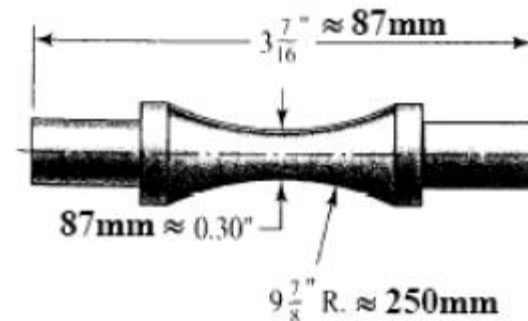
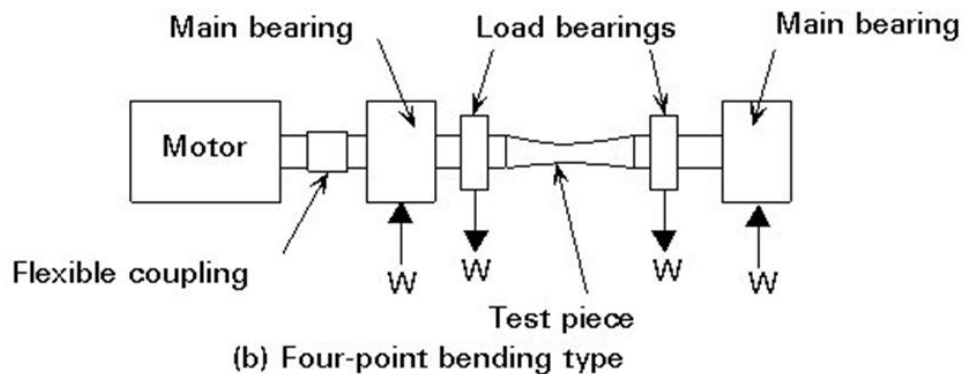
(b) The cantilever rotating bending

(c) The alternating uniaxial tension/compression

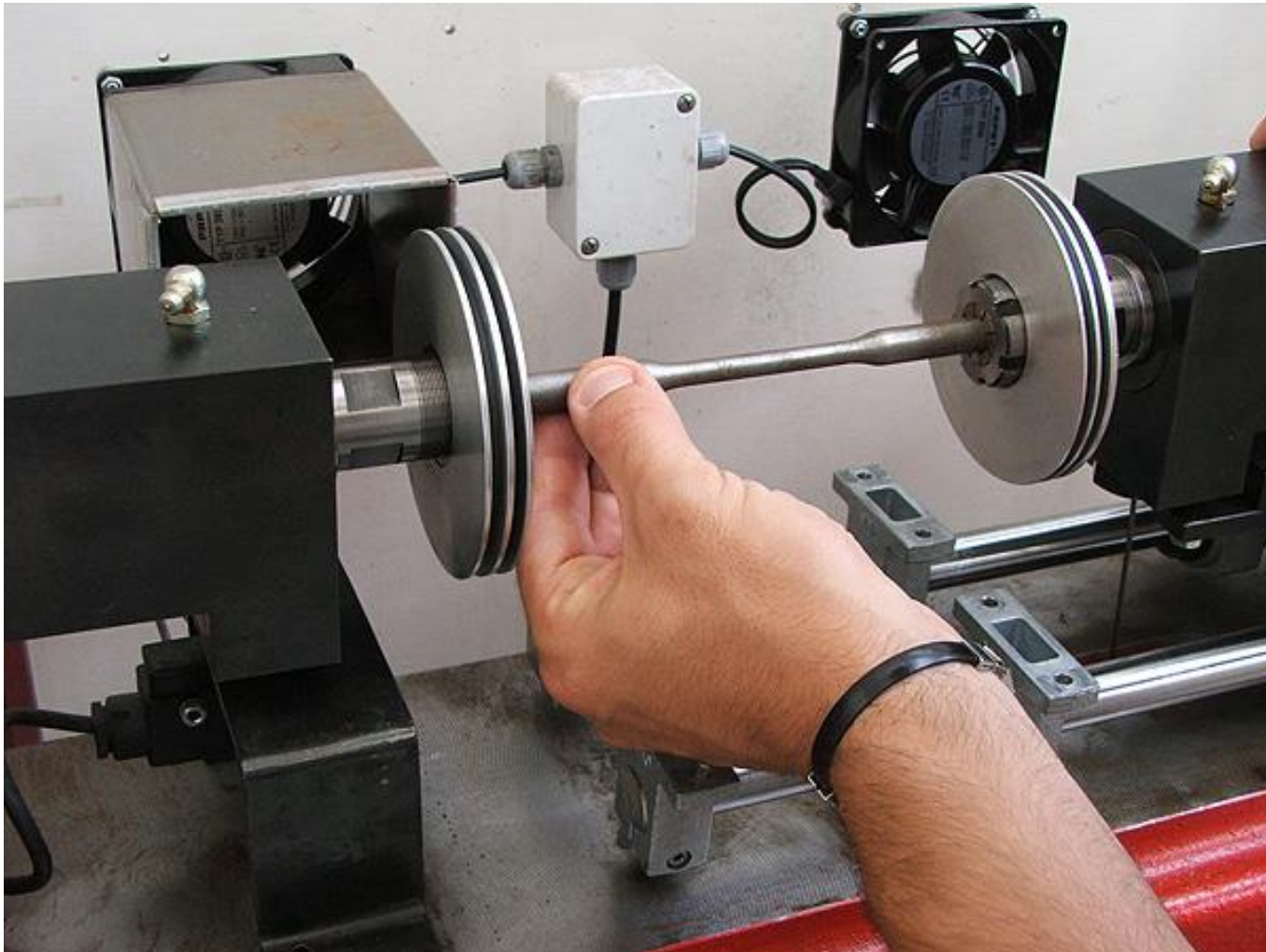
Fatigue test apparatus



(a) Cantilever type



4-point bending apparatus



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watching fatigue test

1

2

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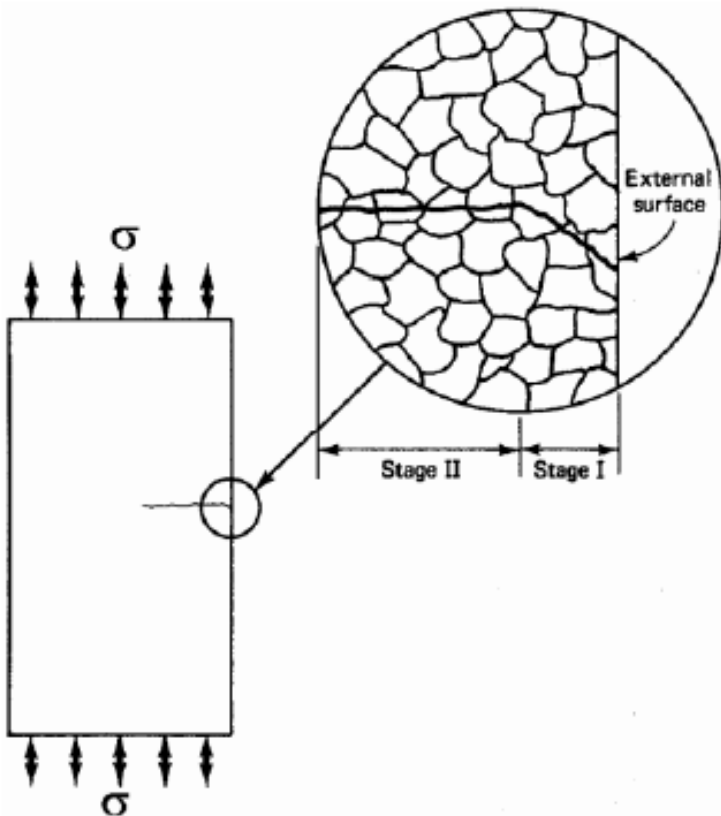
Theory of fatigue failure

Fatigue failures are often referred to as progressive fractures.

For this reasons, this failure is characterized by three stages: *(1) crack initiation*, *(2) crack propagation*, and *(3) fast fracture*.

Initially, fatigue cracks form due to as the result of two stages. *Stage I* that often referred to crack initiation. *Stage II* that often referred to crack growth or the propagation stage. These two stages of fatigue crack are shown schematically in the given figure for a polycrystalline material subjected to an alternating tensile stress in the axial direction.

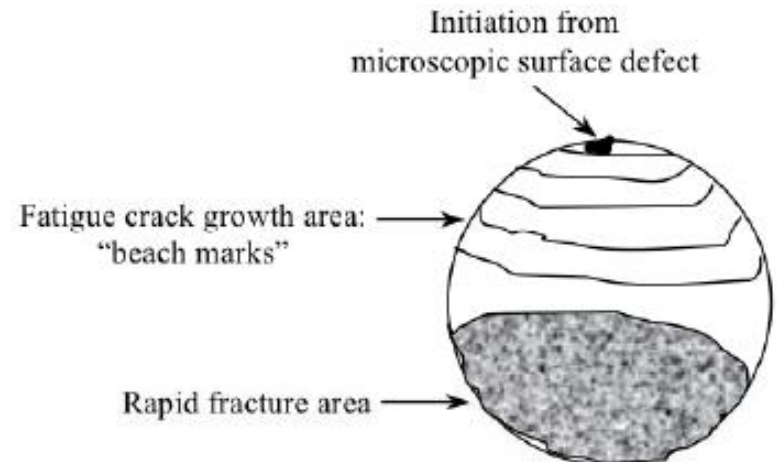
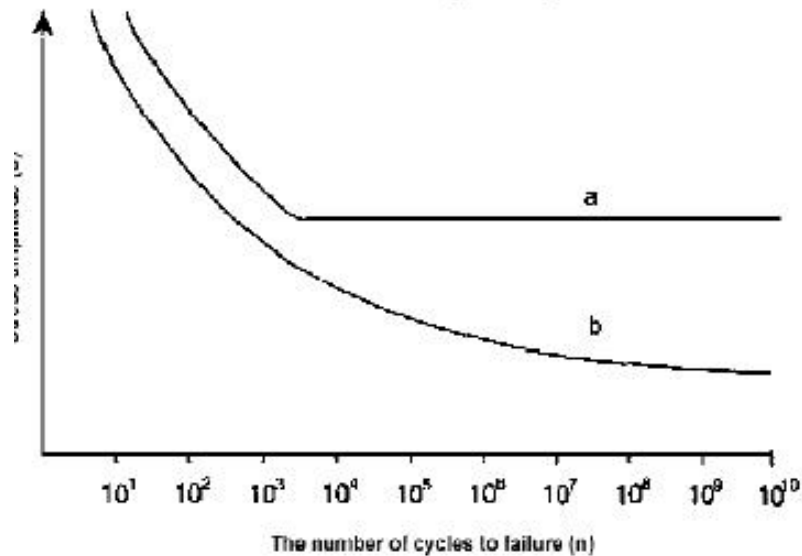
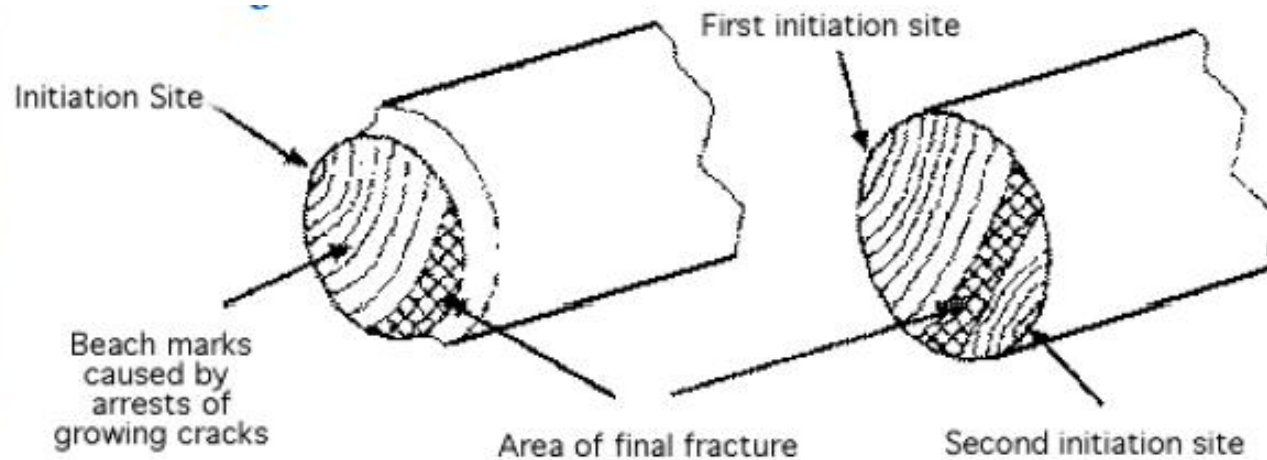
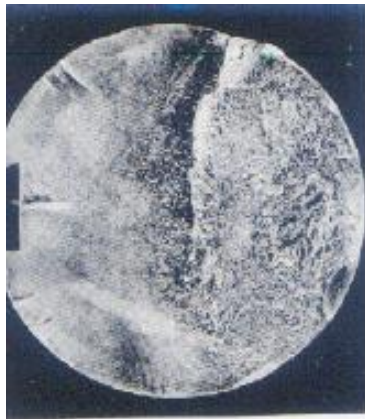
Theory of fatigue failure



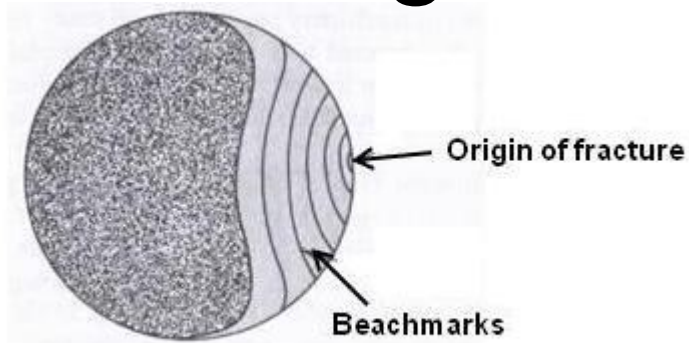
The figure illustrates the cracking as *transgranular* (through the grains) in both stages. This is the common mode of failure, unless the grain boundaries are extremely susceptible to cracking.

The relative number of cycles involved in *stage I* (initiation) and *stage II* (propagation) depends primarily on the stress level. At low stress levels many cycles are required to develop and initiate a crack. As the stress level increases, the crack initiation phase (N_i) decreases.

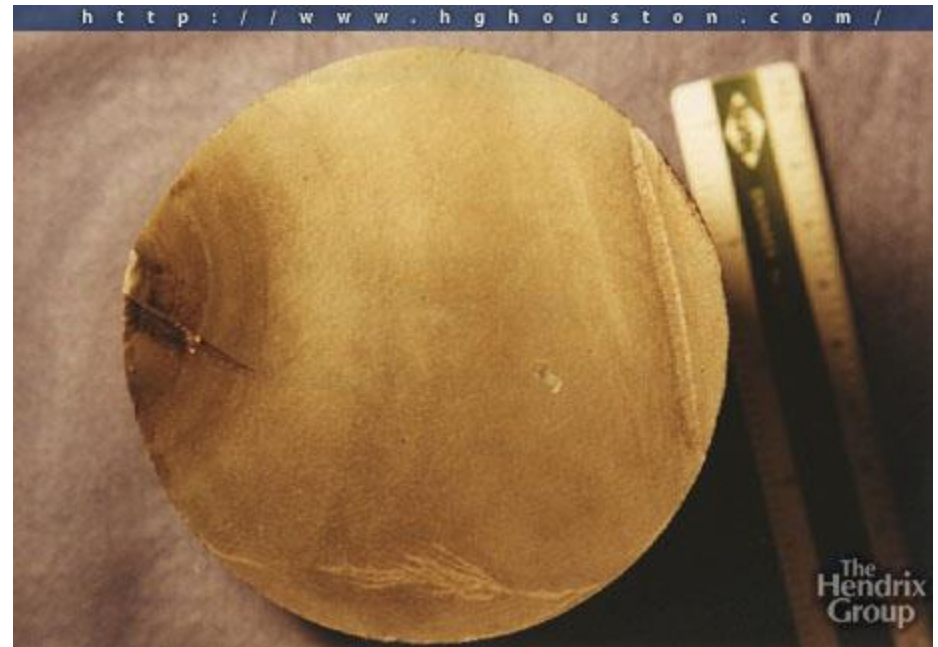
Fatigue mode of failure



Fatigue mode of failure



Fatigue Fracture with Beachmarks



Fatigue mode of failure

Failure is the end result of a process involving the initiation and growth of a crack, usually at the site of a stress concentration on the surface.

Occasionally, a crack may initiate at a fault just below the surface.

Eventually the cross sectional area is so reduced that the component ruptures under a normal service load, but one at a level which has been satisfactorily withstood on many previous occasions before the crack propagated.

The final fracture may occur in a ductile or brittle mode depending on the characteristics of the material.

Fatigue fractures have a characteristic appearance which reflects the initiation site and the progressive development of the crack front, culminating in an area of final overload fracture.

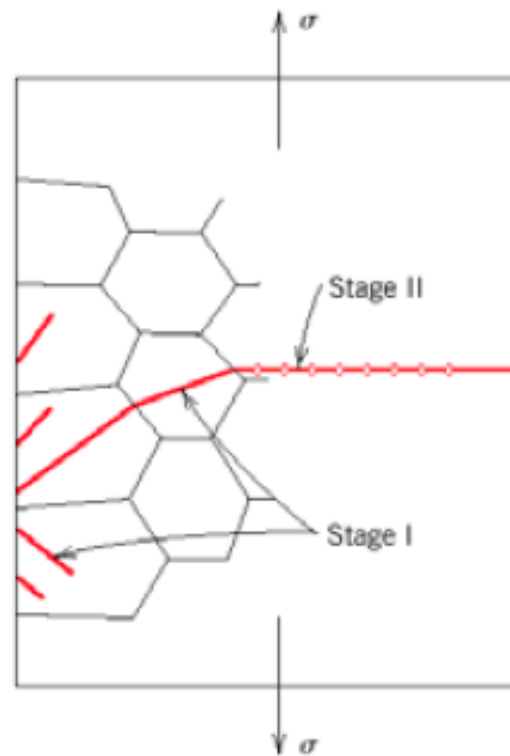
“Failure Mode” illustrates fatigue failure in a circular shaft.

The initiation site is shown and the shell-like markings, often referred to as beach markings because of their resemblance to the ridges left in the sand by retreating waves, are caused by arrests in the crack front as it propagates through the section.

The hatched region on the opposite side to the initiation site is the final region of ductile fracture. Sometimes there may be more than one initiation point and two or more cracks propagate. This produces features as with the final area of ductile fracture being a band across the middle.

Fatigue: Crack initiation and propagation (II)

- Crack initiation at the sites of stress concentration (microcracks, scratches, indents, interior corners, dislocation slip steps, etc.). Quality of surface is important.
- Crack propagation
 - Stage I: initial slow propagation along crystal planes with high resolved shear stress. Involves just a few grains, and has flat fracture surface
 - Stage II: faster propagation perpendicular to the applied stress. Crack grows by repetitive blunting and sharpening process at crack tip. Rough fracture surface.
- Crack eventually reaches critical dimension and propagates very rapidly

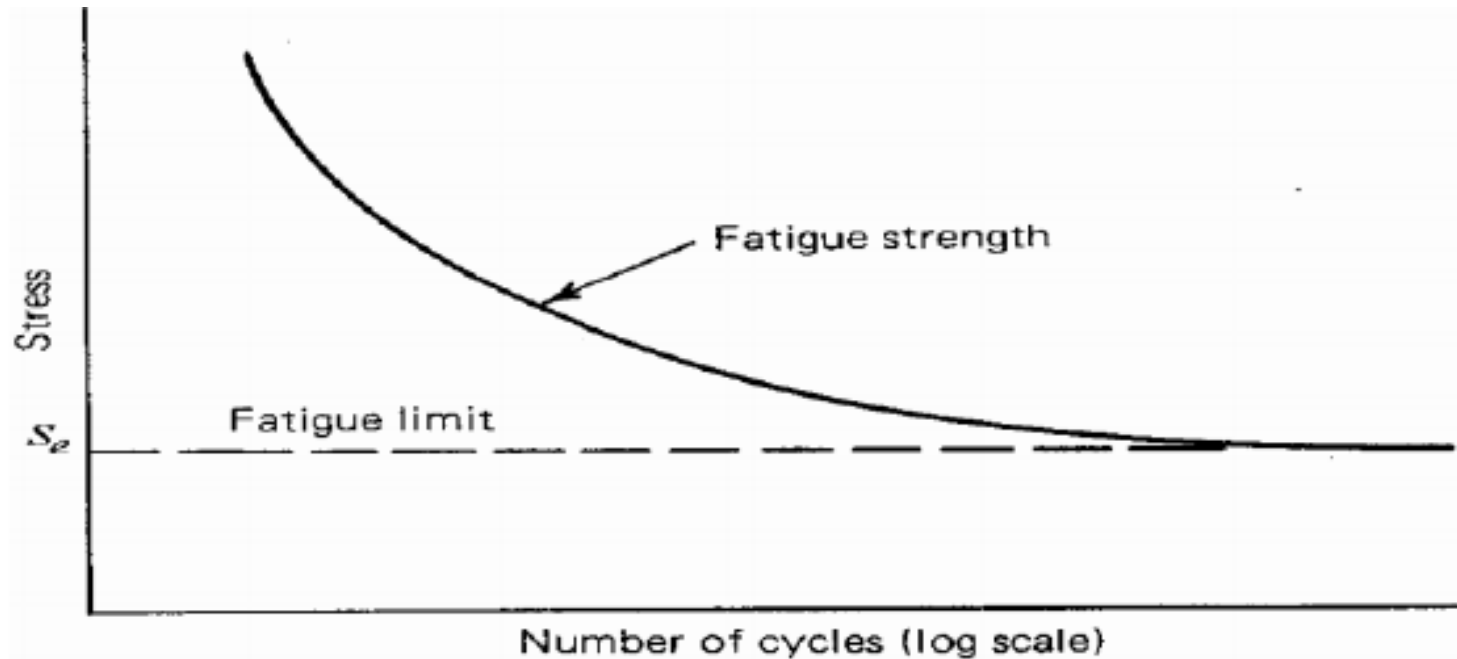


Fatigue strength

The stress at which a metal fails by fatigue testing after a certain number of cycles is termed the *fatigue strength*.

There is a *limiting stress* below which a load may be repeatedly applied an indefinitely large number of times without causing failure. This limiting stress is called the *endurance limit* or *fatigue limit*, (S_e). Its magnitude depends on the kind of stress variation to which the material is subjected. It has to be mentioned that the *fatigue limit* is usually understood to be that for completely reversed stress fatigue

S-N curve



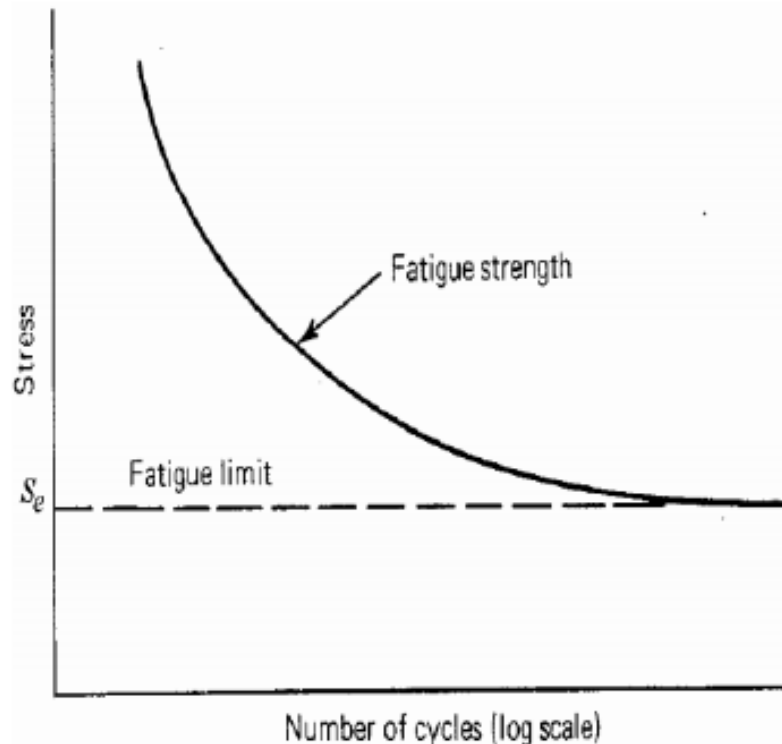
These curves simply indicate the total number of cycles to failure at a given stress level.

Se: endurance limit

the maximum stress of a completely reversed stress cycle that may be applied on the specimen for infinite number of cycles without fatigue failure.

Fatigue strength

Most of the fatigue data, i.e. endurance limit or fatigue limit S_e , which are given in the literature have been determined for conditions of completely reversed cycles of stress where $\sigma_m = 0$. Such data can only inform the designer to keep his design fatigue stress below the curve shown in the S-N curve corresponding to specific material if the mean stress (σ_{mean}) due to fatigue equals to zero.



However, in case that:

$$\sigma_{mean} > \text{zero},$$

which is almost the general case, it will be necessary to construct a diagram, which can be used to define the limit range of maximum (σ_{max}) and minimum (σ_{min}) stresses that could have nonzero mean stress, without taking place fatigue failure. This diagram has been constructed many years ago by *Goodman and Johnson* and many modifications have been made until *Smith* introduce his diagram, which is called *fatigue strength diagram*

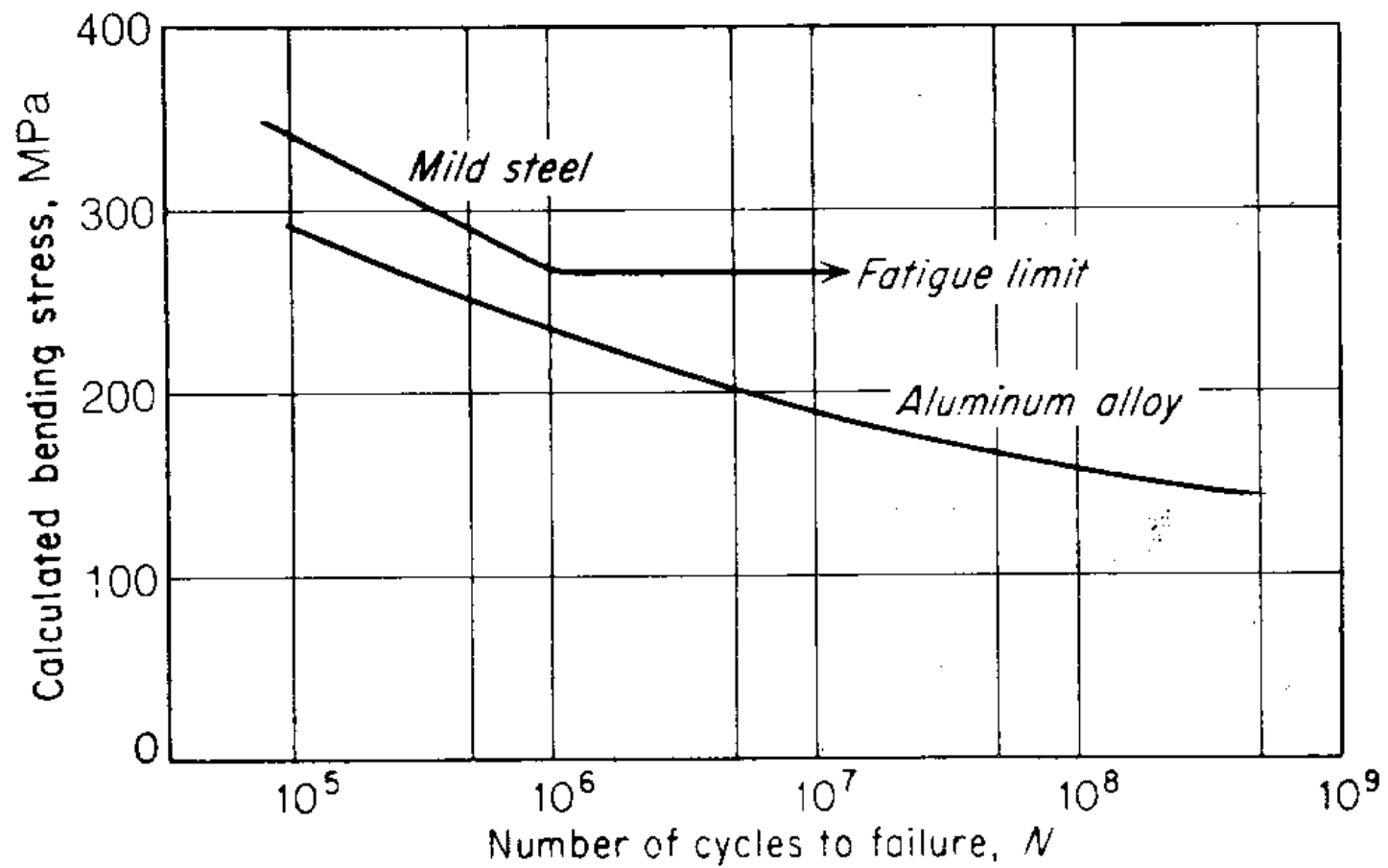


Figure 12-3 Typical fatigue curves for ferrous and nonferrous metals.

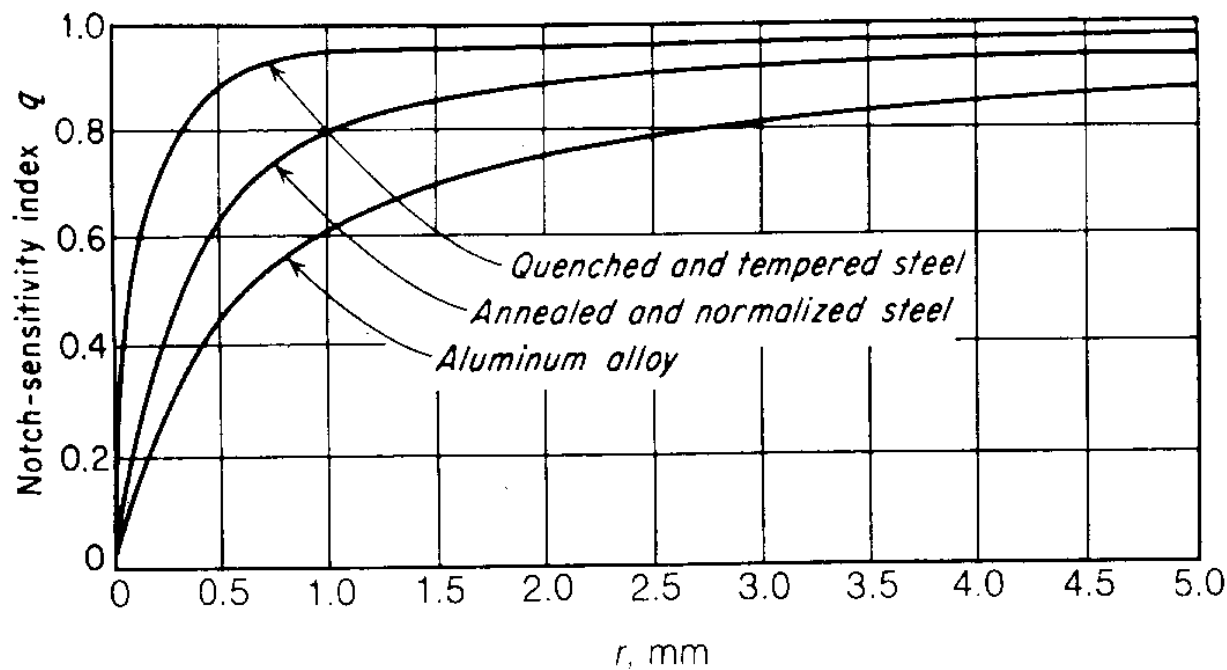
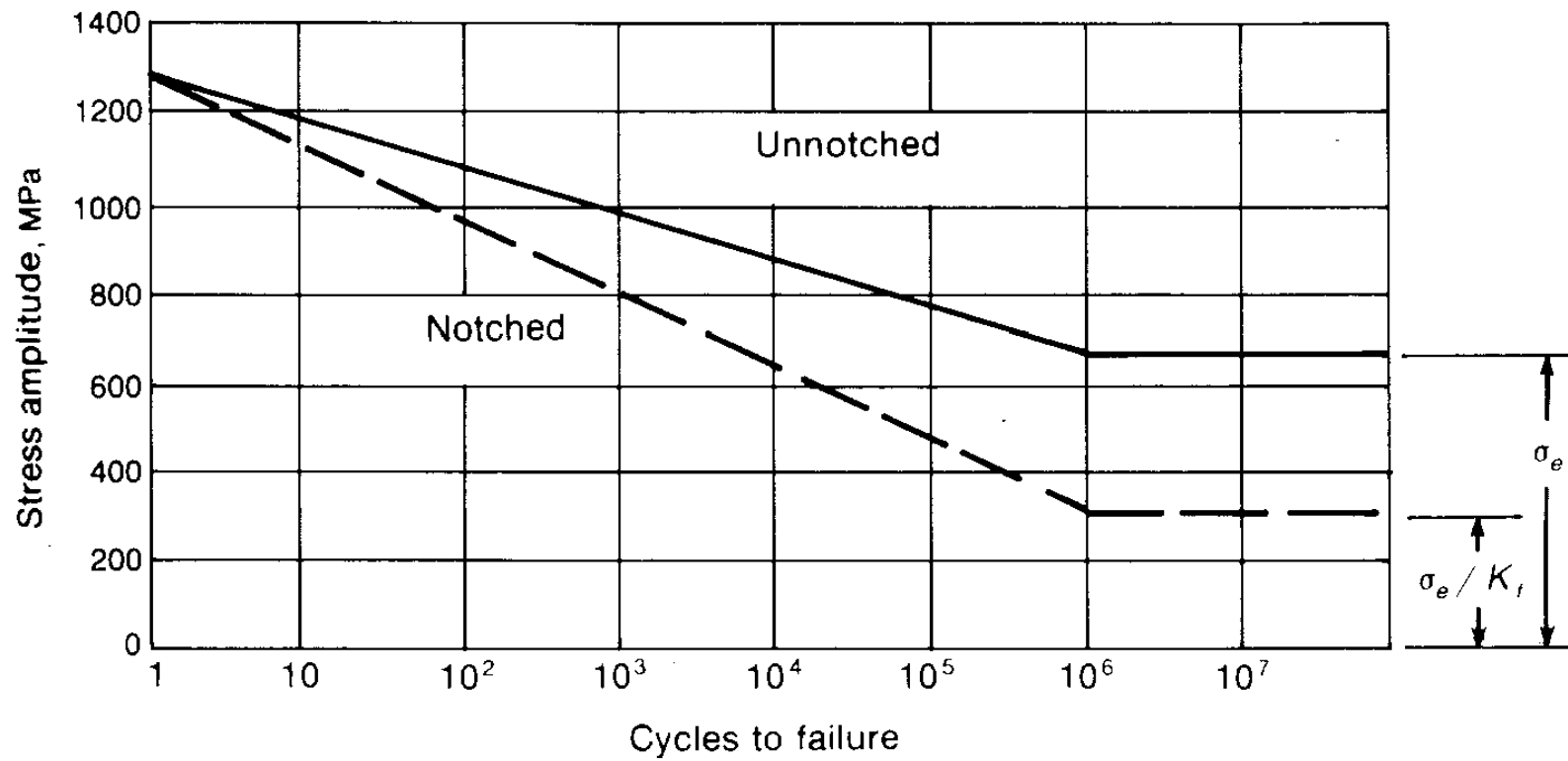


Figure 12-19 Variation of notch-sensitivity index with notch radius for materials of different tensile strength. (After R. E. Peterson, in G. Sines and J. L. Waisman (eds.), "Metal Fatigue," p. 301, McGraw-Hill Book Company, New York, 1959. By permission of the publishers.)



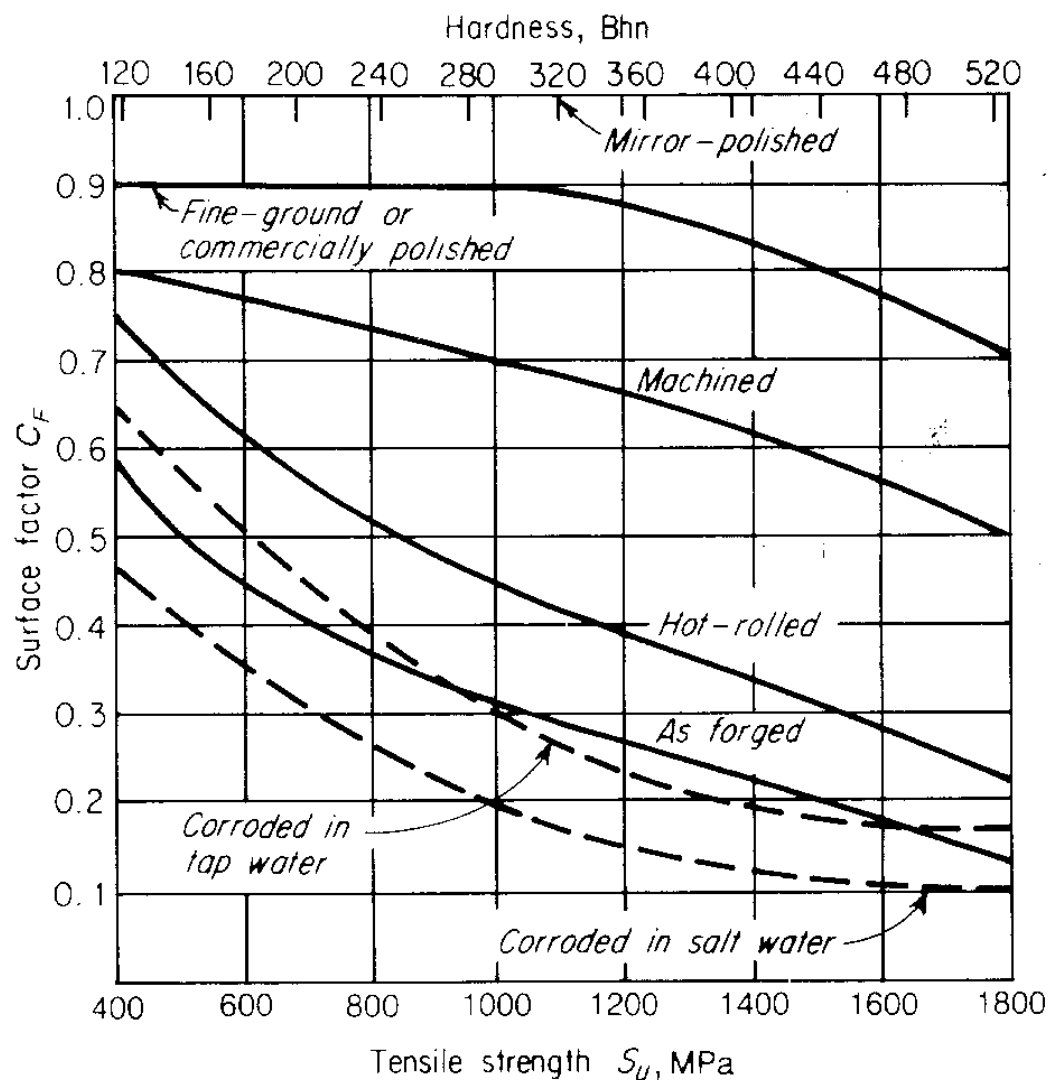


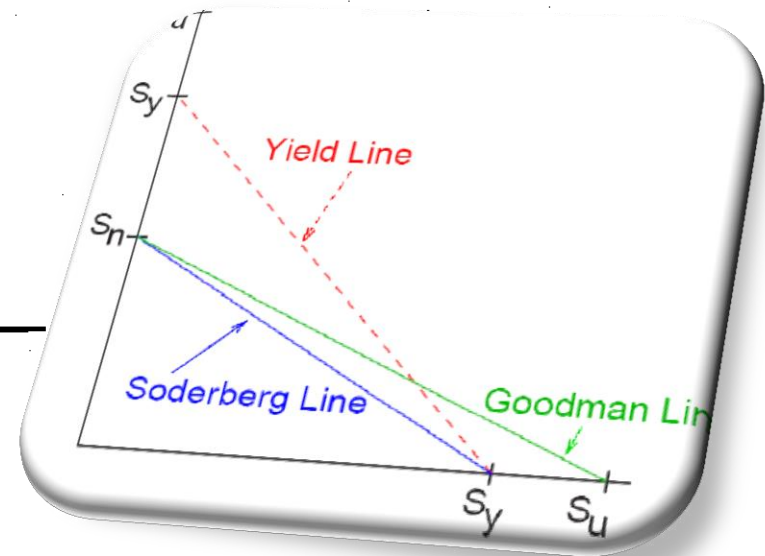
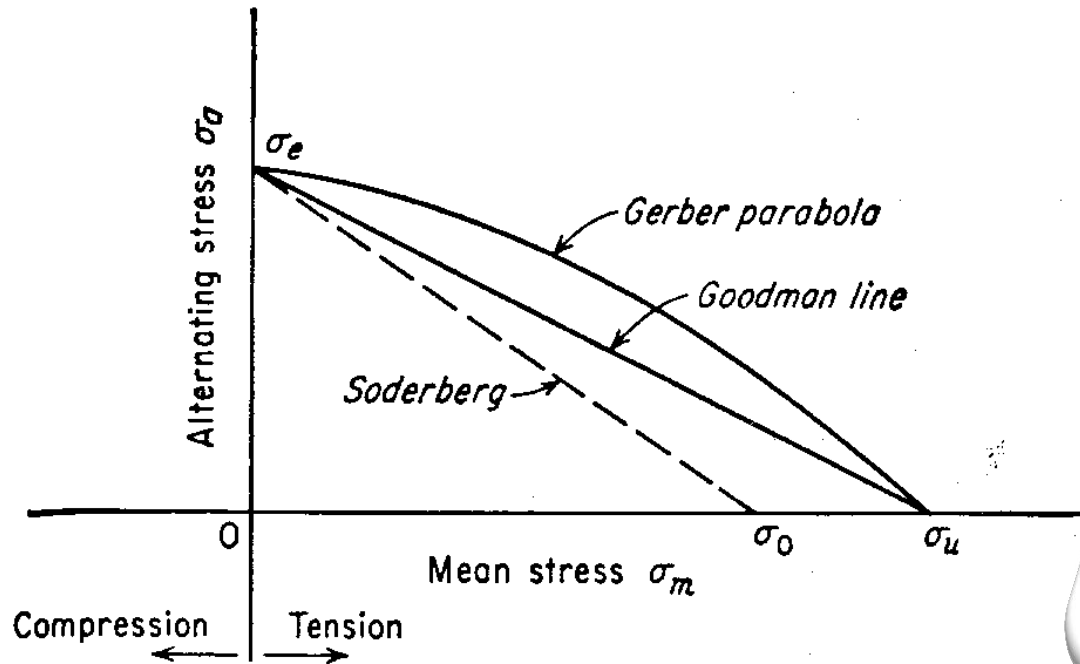
Figure 12-20 Reduction factor for fatigue limit of steel due to various surface treatments. (After R. C. Juvinall, "Stress, Strain, and Strength," p. 234, McGraw-Hill Book Company, New York, 1967. By permission of the publishers.)

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Soderberg & Goodman lines

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Soderberg & Goodman diagrams



Soderberg Line is based on S_y and S_n :

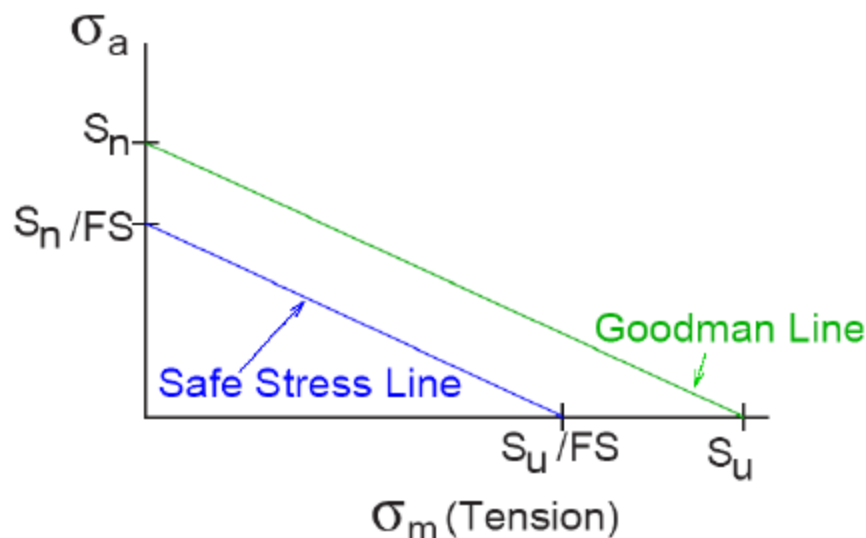
$$\sigma_a/S_n + \sigma_m/S_y = 1$$

Goodman Line is based on S_u and S_n :

$$\sigma_a/S_n + \sigma_m/S_u = 1$$

Factor of Safety

Illustrated here for
Goodman Criteria
(can use Soderberg, etc., also).



Note: Factor of Safety may be different for each mechanical property

Equations

Soderberg Line is based on S_y and S_n :

$$S_a/(S_n/FS) + S_m/(S_y/FS) = 1$$

$$n_f = \frac{S_e S_{yt}}{\sigma_a S_{yt} + \sigma_m S_e}$$

Goodman Line is based on S_u and S_n :

$$S_a/(S_n/FS) + S_m/(S_u/FS) = 1$$

$$n_f = \frac{S_e S_{ut}}{\sigma_a S_{ut} + \sigma_m S_e}$$

Solved Problem

Estimate the factor of safety guarding against fatigue and first-cycle yielding using:

(a). Soderberg criterion

(b). Modified Goodman criterion

$$S_{ut}=590\text{MPa}, S_y=490\text{MPa}, \sigma_a=280\text{MPa}, \sigma_m=140\text{MPa}, S_e=280\text{MPa}$$

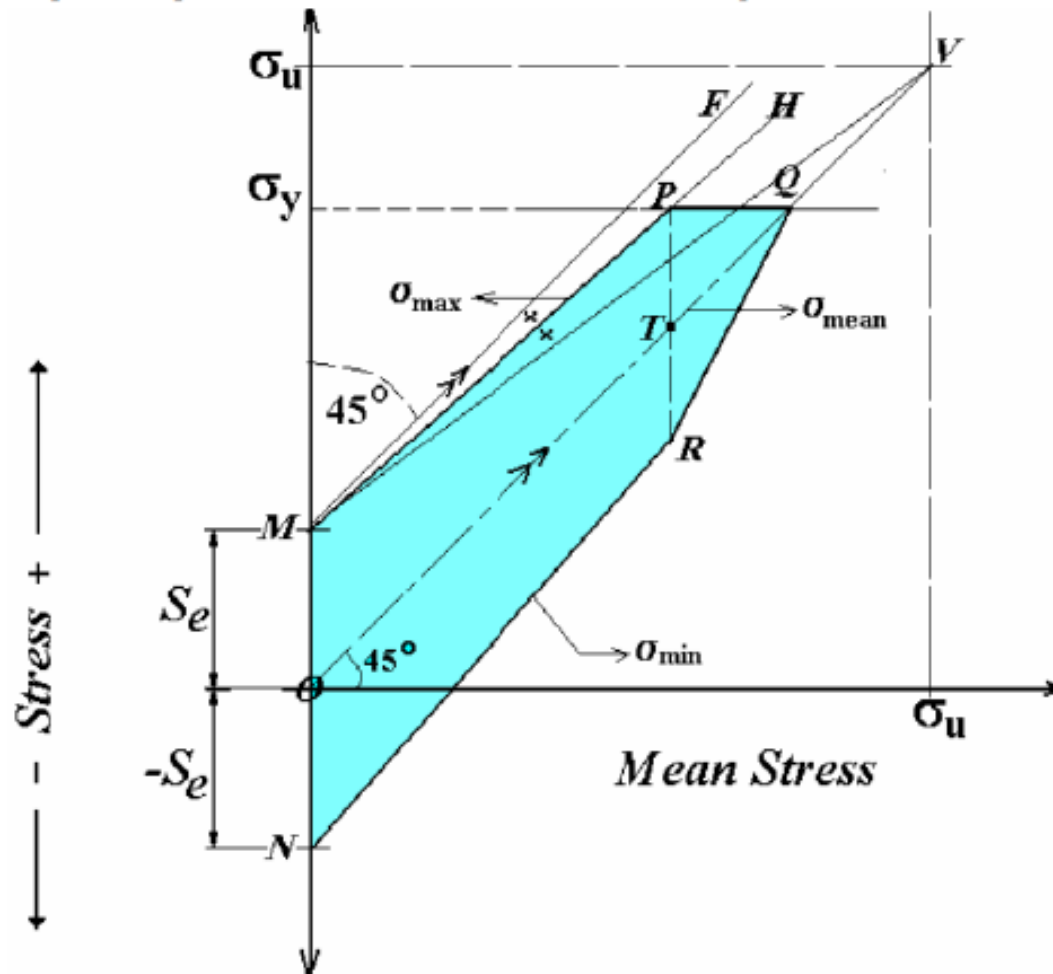
Criteria	Fatigue factor of safety, n_f	Answer
1 st -yielding cycle	$n_y = \frac{S_y}{\sigma_a + \sigma_m}$	1.17
Soderberg	$n_f = \frac{S_e S_{yt}}{\sigma_a S_{yt} + \sigma_m S_e}$	0.8
Modified Goodman	$n_f = \frac{S_e S_{ut}}{\sigma_a S_{ut} + \sigma_m S_e}$	0.81
Gerber	$n_f = \frac{1}{2} \left(\frac{S_{ut}}{\sigma_m} \right)^2 \left(\frac{\sigma_a}{S_e} \right) \left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut} \sigma_a} \right)^2} \right]$	0.95

Solve this Problem

Using Soderberg and Goodman lines, check the repeating stress cycle with maximum stress = 120 MPa if $S_n=180\text{MPa}$, $S_y=360\text{MPa}$, $UTS=520\text{MPa}$, $FS=1.5$ (in tension) and $FS = 2$ in fatigue.

Smith diagram

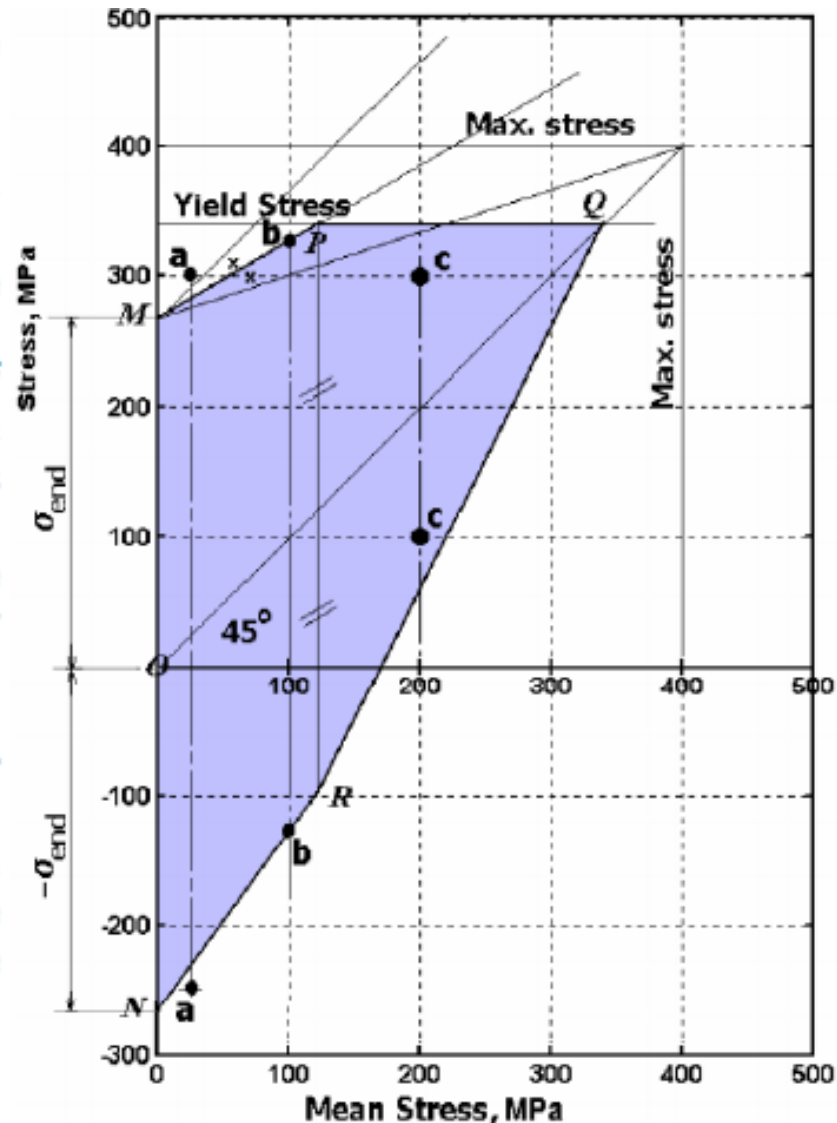
The subsequent procedure are followed to plot the smith diagram:



Smith diagram

Location **a** is subjected to fatigue stress outside the bounded area **MPQRN**, which mean that the fracturing due to fatigue is expected. Location **b** is subjected to almost fatigue stress on the contour of the bounded area **MPQRN**, which mean the risk of fatigue fracturing. The location **c** is subjected to fatigue stress inside the bounded area **MPQRN**, which mean that fracturing due to fatigue life is completely unlikely at this location.

This rod must be replaced after certain service lifetime and before fracturing at location **a** and **b**. Otherwise, material type or the part dimensions should be changed, such that the applied stresses are inside the bounded area **MPQRN**.



$$\sigma_a = \sigma_e \left[1 - \left(\frac{\sigma_m}{\sigma_u} \right)^x \right]$$

Example A 4340 steel bar is subjected to a fluctuating axial load that varies from a maximum of 330 kN tension to a minimum of 110 kN compression. The mechanical properties of the steel are:

$$\sigma_u = 1090 \text{ MPa} \quad \sigma_0 = 1010 \text{ MPa} \quad \sigma_e = 510 \text{ MPa}$$

Determine the bar diameter to give infinite fatigue life based on a safety factor of 2.5.

Since the bar has a constant cylindrical cross section A , the variation in stress will be proportional to the load

$$\begin{aligned} \sigma_{\max} &= \frac{0.330}{A} \text{ MPa} & \sigma_{\min} &= -\frac{0.110}{A} \text{ MPa} \\ \sigma_m &= \frac{\sigma_{\max} + \sigma_{\min}}{2} = \frac{0.330/A + (-0.110/A)}{2} = \frac{0.110}{A} \text{ MPa} \\ \sigma_a &= \frac{\sigma_{\max} - \sigma_{\min}}{2} = \frac{0.330/A - (-0.110/A)}{2} = \frac{0.220}{A} \text{ MPa} \end{aligned}$$

using the conservative Goodman line and Eq. (12-7)

$$\begin{aligned} \sigma_a &= \sigma_e \left(1 - \frac{\sigma_m}{\sigma_u} \right) & \sigma_e &= \frac{510}{2.5} = 204 \text{ MPa} \\ \frac{0.220/A}{204} &= 1 - \frac{0.110/A}{1090} \\ A &= \frac{0.220}{204} + \frac{0.110}{1090} = (1.078 + 0.101) \times 10^{-3} \text{ m}^2 = 1179 \text{ mm}^2 \\ D &= \sqrt{\frac{4A}{\pi}} = 38.7 \text{ mm} \end{aligned}$$